C^*-algebras of Bergman type operators with piecewise continuous coefficients and shifts

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Let U be a bounded simply connected domain in $\mathbb{C}$ with sufficiently smooth boundary Γ, let G be a commutative group of conformal mappings of $\overline{U}$ onto itself which is similar to the group of elliptic, hyperbolic or parabolic mappings of the closed unit disc $\overline{\mathbb{D}}$ onto itself, and let $\mathfrak{L}_G$ be a G-invariant set of simple Liapunov curves in $\overline{U}$ such that for every $z \in \overline{U}$ an at most finite number of curves in $\mathfrak{L}_G$ are intersecting at z and at every $z \in \Gamma$ these curves form with Γ pairwise distinct angles lying in $(0, \pi)$. Let $\mathfrak{A}_{n,m}(U, \mathfrak{L}_G)$ be the C^*-algebra generated by n poly-Bergman projections, m anti-poly-Bergman projections, and by all multiplication operators aI acting on the space $L^2(U)$, where a are piecewise continuous functions on $\overline{U}$ with possible discontinuities on $\mathfrak{L}_G$ that are continuous at common fixed points of $g \in G$. For mentioned groups G, applying a local-trajectory method and a Fredholm symbol calculus for the C^*-algebra $\mathfrak{A}_{n,m}(U, \mathfrak{L}_G)$, we establish Fredholm criteria for the operators B in the C^*-algebras $\mathfrak{B}$ generated by all operators $A \in \mathfrak{A}_{n,m}(U, \mathfrak{L}_G)$ and all weighted shift operators W_g $(g \in G)$, where $W_g f = g'(f \circ g)$ for $f \in L^2(U)$.