The Boundedness and compactness of a class of h-Fourier integral operators

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Abstract

For $\varphi \in \mathcal{S}(\mathbb{R}^n)$ (the Schwartz space), the integral operators

$$F_h\varphi(x) = \iint e^{\frac{i}{h}(S(x,\theta)-y\theta)} a(x,\theta)\,\varphi(y)\,dyd\theta \tag{1}$$

appear naturally in the expression of the solutions of the semiclassical hyperbolic partial differential equations and in the expression of the C^∞-solution of the associate Cauchy's problem. Which appear two C^∞-functions, the phase function $\phi(x,y,\theta) = S(x,\theta) - y\theta$ and the amplitude a..

In this work, we apply the technique of [1] to establish the boundedness and the compactness of the operators (1). To this end we give a brief and simple proof for a result of [1] in our framework.

We mainly prove the continuity of the operator F_h on $L^2(\mathbb{R}^n)$ when the weight of the amplitude a is bounded. Moreover, F_h is compact on $L^2(\mathbb{R}^n)$ if this weight tends to zero. Using the estimate given in [4] for h-pseudodifferential (h-admissible) operators, we also establish an L^2-estimate of $\|F_h\|$.

We note that if the amplitude a is juste bounded, the Fourier integral operator F_h is not necessarily bounded on $L^2(\mathbb{R}^n)$. Recently, M. Hasanov [2] and B. Messirdi-A. Senoussaoui [3] constructed a class of unbounded Fourier integral operators with an amplitude in the Hörmander's class $S^0_{1,1}$ and in $\bigcap\limits_{0<\rho<1} S^0_{\rho,1}$.

Keywords: h-Fourier integral operators, h-pseudodifferential operators, symbol and phase.

References

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