CONSTRAINED VON NEUMANN INEQUALITIES, HIGHER RANK NUMERICAL RANGE AND APPLICATION TO HARMONIC ANALYSIS

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Let denote by $S(\phi)$ the extremal operator defined by the compression of the unilateral shift S to the model subspace $H(\phi) = \mathbb{H}^2 \ominus \phi\, \mathbb{H}^2$ as the following $S(\phi)f(z) = P(zf(z))$, where P denotes the orthogonal projection from the Hardy space $\mathbb{H}^2$ onto $H(\phi)$ and ϕ is an inner function on the unit disc. The numerical radius seems to be important and have many applications in harmonic analysis like the following theorem wich gives an extension of a previous result of C. Badea and G. Cassier [1].

Theorem 0.1 ([7] Theorem 2.1). *Let $F = P/Q$ be a rational function which is positive on the torus, where P and Q are coprime. Denote by*

$$\phi(z) = \prod_{j=1}^{p} \left(\frac{z - \alpha_j}{1 - \overline{\alpha_j} z} \right)^{m_j}$$

and

$$\psi(z) = \prod_{j=1}^{q} \left(\frac{z - \beta_j}{1 - \overline{\beta_j} z} \right)^{d_j}$$

the respectively finite Blashke products formed by the nonzero roots of P and Q in the open disc, let $m = \sum_{j=1}^{p} m_j$ and $d = \sum_{j=1}^{q} d_j$. Then the Taylor coefficient c_k of order k of F satisfies the following inequality:

$$|c_k| \leqslant c_0 \ \omega_2({S^*}^k(\varphi)), \ \textit{where } \varphi(z) = z^{\max(0, m-d+1)} \psi(z).$$

In this talk, we give an explicit formula of the numerical radius of $S(\phi)$ in the particular case where ϕ is a finite Blaschke product with unique zero and an estimate on the general case. We establish also a sharpened Schwarz-Pick operatorial inequality generalizing a U. Haagerup and P. de la Harpe result for nilpotent operators [6].

The second part is devoted to the study of the higher rank-k numerical range denoted by $\Lambda_k(T)$ which is the set of all complex number λ satisfying $PTP = \lambda P$ for some rank-k orthogonal projection P. This notion was introduced by M.-D. Choi, D. W. Kribs, et K. Zyczkowski motivated by a problem in Physics. We show that if S_n is the n-dimensional shift then its rank-k numerical range is the circular disc centered in zero and with a precise radius.

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