



Program of the  
*Eleventh Advanced Course*  
*in Operator Theory*  
*and Complex Analysis*

Sevilla, June 16th-18th, 2014



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## Schedule

**Main courses** will be held with the following timetable.

### MONDAY - TUESDAY - WEDNESDAY

|              |                 |
|--------------|-----------------|
| 09:00 -09:50 | N. ARCOZZI      |
| 10:00 -10:50 | A. OLEVSKII     |
| 11:00 -11:30 | Coffee Break    |
| 11:30 -12:20 | J.R. PARTINGTON |
| 12:30 -13:20 | A. ULANOVSKII   |

We will have **Lunch Break from 13:30 to 15:00.**

**Plenary lectures and Contributed talks** will be distributed as follows.

### MONDAY

|               |                |
|---------------|----------------|
| 15:00 - 16:00 | A. POLTORATSKI |
| 16:00 - 16:30 | D. ESTÉVEZ     |
| 16:30 -16:50  | Coffee Break   |
| 16:50 - 17:20 | A. HAIMI       |
| 17:20 - 17:50 | S. HAMOUDA     |
| 17:50 - 18:20 | J.C. SÁNCHEZ   |

### TUESDAY

|               |                 |
|---------------|-----------------|
| 15:00 - 16:00 | I. CHALENDAR    |
| 16:00 - 16:30 | B. ADEL         |
| 16:30 -16:50  | Coffee Break    |
| 16:50 - 17:20 | A. AL-RAWASHDEH |
| 17:20 - 17:50 | A. BADREDDINE   |
| 17:50 - 18:20 | D. CARDONA      |

### WEDNESDAY

|               |              |
|---------------|--------------|
| 15:00 - 15:30 | C. CHERIFA   |
| 15:30 - 16:00 | G. DE SOUZA  |
| 16:00 - 16:30 | M. SARIH     |
| 16:30 - 17:00 | D. ISRAFILOV |
| 17:00 - 17:30 | A. GHERBI    |

# Abstracts

## **Courses**

**09:00 – 09:50 N. Arcozzi**

(Bologna University, Italy)

### **Some discrete models in complex analysis.**

**Abstract.** It is often fruitful to work on a discrete (e.g. dyadic) analog of a problem originally posed in a different context. Sometimes, the solution of the discrete analog directly leads to the solution of the original problem. In these lectures we will see some examples related to analytic function spaces.

- (i) One complex dimension: Carleson measures for the analytic Dirichlet space vs. weighted inequalities on trees.
- (ii) Several complex dimensions: multipliers for the Drury-Arveson space vs. weighted inequalities on quotient structure for trees.
- (iii) Potential theory on the real line and beyond: classical capacity vs. capacity on the trees' boundary.

**10:00 – 10:50 A. Olevskii**

(Tel Aviv University, Israel)

**Sampling of signals and exponential frames.**

**Abstract.** How often one should measure a signal with given spectrum in order to be able to recover it ? Does a "universal" sampling exist , which works well for any spectrum of fixed size, independently of its structure and localization ? I will survey classical background and discuss the recent progress.

**11:30 – 12:20 J.R. Partington**

(University of Leeds, U.K.)

**Inner functions and operator theory.**

**Abstract.** Inner functions arose originally in an operator-theoretic context, via Beurling's characterization of the invariant subspaces of the unilateral shift. Since then, they have played important roles in a number of operatorial contexts, which we shall survey. This includes the theory of composition operators on Hardy spaces, the theory of model spaces and restricted shifts, and certain interesting questions about interpolation by analytic functions.



**12:30 – 13:20 A. Ulanovskii**

(University of Stavanger, Norway)

**Sampling and interpolation problems in several dimensions.**

**Abstract.** There is a fundamental difference between sampling and interpolation of functions of one and several variables: in dimension  $n = 1$  the zeros of an entire function  $f$  are discrete, and there is a precise connection between the asymptotic behavior/density of the zeros and the growth of  $f$ . It is no longer true in several dimensions. As a consequence, much is known about the interpolation and sampling of bandlimited functions in one dimension, but only a few results are known in higher dimensions. We give a short introduction into the subject and present new sharp sufficient conditions for interpolation and sampling for functions of  $n$  variables with convex spectrum.

## Plenary lectures and contributed Talks

MONDAY

**15:00 – 16:00 A. Poltoratski**

(Texas A&M University, USA)

*Spectral Gaps and Polya Sequences.*

**Abstract.** We discuss the gap problem, the problem of estimating the size of a gap in the support of the Fourier transform of a measure on the real line, within general scope of Uncertainty Principle in Harmonic Analysis. We consider applications of the gap problem to adjacent problems of complex and harmonic analysis. This talk is partially based on joint work with M. Mitkovski.

## 16:00 – 16:30 D. Estévez

(Universidad Autónoma de Madrid, Spain.)

### *Separation of singularities, generation of algebras and complete $K$ -spectral sets.*

**Abstract.** In this talk, we will show a certain relation between the generation of uniform analytic algebras and complete  $K$ -spectral sets of Hilbert space operators. We also show the relation with earlier results on separation of singularities of bounded analytic functions by Havin, Nersessian and Cerdá.

Havin and Nersessian showed that, under certain geometric conditions on domains  $\Omega_1, \Omega_2$  in  $\mathbb{C}$ , every function  $f \in H^\infty(\Omega_1 \cap \Omega_2)$  can be written as  $f = f_1 + f_2$ , with  $f_j \in H^\infty(\Omega_j)$ . This result can be seen as a separation of singularities. This problem can be related to that of writing an  $f \in H^\infty(\Omega)$  as  $f = f_1 \circ \varphi_1 + f_2 \circ \varphi_2$ , where  $\varphi_1$  and  $\varphi_2$  are fixed analytic functions in  $\Omega := \Omega_1 \cap \Omega_2$ . More generally, one can ask whether  $f$  can be written as a sum of products of functions of the form  $g \circ \varphi_j$ ,  $j = 1, 2$ . We will study geometric conditions that guarantee that every function in  $H^\infty(\Omega)$  can be written in this way. We also have analogous results for finite collections  $\{\varphi_1, \dots, \varphi_n\}$  of any size and for the algebra  $A(\overline{\Omega})$  of functions analytic in  $\Omega$  and continuous in  $\overline{\Omega}$ . Whenever our conditions hold,  $\{\varphi_1, \dots, \varphi_n\}$  are generators of the algebra  $H^\infty(\Omega)$  (and of the algebra  $A(\overline{\Omega})$ ) in the norm topology.

Next we apply these results to studying complete  $K$ -spectral sets. Let  $T$  be an operator on a Hilbert space  $H$ . A compact subset  $X$  of  $\mathbb{C}$  is said to be a complete  $K$ -spectral set for  $T$  if  $\|f(T)\|_{\mathcal{B}(H \otimes \mathbb{C}^s)} \leq K \sup_{z \in X} \|f(z)\|_{\mathcal{B}(\mathbb{C}^s)}$ , for every  $s \times s$  rational matrix function  $f$  with poles outside of  $X$  of any size  $s \geq 1$ . Complete  $K$ -spectrality in a domain  $\overline{\Omega}$  for an operator  $T$  is equivalent to the similarity of  $T$  to some operator which has a rational normal dilation to  $\partial\Omega$ . We will use our results on algebra generation to give tests for complete  $K$ -spectrality. These will have the form: “if  $\|\varphi_k(T)\| \leq 1$  for every  $k$ , then  $\overline{\Omega}$  is a complete  $K$ -spectral set for  $T$ , for some  $K$ .”

This is joint work with Dmitry Yakubovich (Univ. Autónoma de Madrid) and Michael Dritschel (Newcastle Univ.).

**16:50 – 17:20 A. Haimi**

(NTNU, Norway)

*Polyanalytic Bergman Kernels.*

**Abstract.** We discuss Hilbert spaces of polyanalytic functions on the complex plane. The norm on these spaces is given by integration against a weight  $e^{-mQ(z)}$  where  $Q$  is a strictly subharmonic function and  $m$  a large positive scaling parameter. We obtain a near-diagonal asymptotic expansion for the reproducing kernels as  $m$  tends to infinity. In the setting of one complex variable, this generalizes the work of Tian-Yau-Catlin-Zelditch from analytic functions to polyanalytic functions.

We also study reproducing kernels of corresponding polynomials spaces. These are spanned by functions  $\bar{z}^r z^j$  where  $0 \leq r \leq q-1$  and  $0 \leq j \leq n-1$ . The inner product is induced by the same weight as before. Keeping  $q$  fixed and letting  $n$  and  $m$  go to infinity, we obtain scaling limits for the kernels in so called bulk regime. In the model case  $Q(z) = |z|^2$ , this investigation has applications in statistical quantum mechanics.

Some of the results are joint work with Håkan Hedenmalm.

**17:20 – 17:50 S. Hamouda**

(University of Mostaganem, Algeria)

*Growth of solutions of certain linear differential equations in the complex plane.*

**Abstract.** In this talk, we investigate the growth of solutions of certain linear differential equations by making use the Nevanlinna value distribution theory of meromorphic function on the complex plane (see [?]).

the iterated  $n$ -order of growth of meromorphic function  $f(z)$  is defined by

$$\sigma_n(f) = \limsup_{r \rightarrow \infty} \frac{\log_n T(r, f)}{\log r},$$

where  $T(r, f)$  is the Nevanlinna characteristic function of  $f$ . For an entire function  $f(z)$ , we have

$$\sigma_{M,n}(f) = \sigma_n(f) = \limsup_{r \rightarrow \infty} \frac{\log_{n+1} M(r, f)}{\log r},$$

where  $M(r, f) = \max_{|z|=r} |f(z)|$ .

Here, we use a new approach on the iterated type of the coefficients. This work gives a generalization and an improvement of previous results in this field of research.

**17:50 – 18:20 J.C. Sánchez**

(Universidade do Algarve, Portugal)

*Rational matrix factorization and difference equations.*

**Abstract.** The matrix-function factorization find applications in many fields like diffraction theory, the theory of differential equations and the theory of singular integral operators. However, only for a few classes of matrices is known the explicit formulas for the factors of the factorization.

Our talk will be devoted to the relation between the factorization of rational matrices on the unit circle and the solution of a linear system of difference equations with constant coefficients.

We also will provide some useful examples.

## TUESDAY

**15:00 – 16:00 I. Chalendar**

(Lyon I University, France)

*An extremal problem for characteristic functions.*

**Abstract.** Suppose  $E$  is a subset of the unit circle  $\mathbb{T}$  and  $H^\infty \subset L^\infty$  is the Hardy subalgebra. We examine the problem of finding the distance from the characteristic function of  $E$  to  $z^n H^\infty$ . Precise solutions are given in several important cases. The techniques used involve the theory of Toeplitz and Hankel operators as well as the construction of certain conformal mappings.

**16:00 – 16:30 B. Adel**

(University Abdelmaleek Essaadi, Tanger)

*On generalized Weyl's type theorem.*

**Abstract.** It is shown that if a bounded linear operator  $T$  or its adjoint  $T^*$  has the single-valued extension property, then generalized Browder's theorem holds for  $f(T)$  for every  $f \in \mathcal{H}(\sigma(T))$ . We establish the spectral theorem for the B-Weyl spectrum which generalizes a result in the Hilbert space case and we give necessary and sufficient conditions for such operator  $T$  to obey generalized Weyl's theorem.



**16:50 – 17:20 A. Al-Rawashdeh**

(UAE University, UAE)

*Operator Algebras and Unitary Operators.*

**Abstract.** In operator algebras, one of the main tools which is used in the classification is  $K$ -theory. The group of unitaries is also a good tool. We shall review main results regarding the unitaries. In the case of von Neumann algebras, H. Dye proved that the discrete unitary group in a factor determines the algebraic type of the factor. Afterwards, for a large class of simple unital  $C^*$ -algebras, Al-Rawashdeh, Booth and Giordano proved that the algebras are  $*$ -isomorphic if and only if their unitary groups are isomorphic as abstract groups. The simplicity of the algebra is a necessary condition, as we give a counter example in the non-simple case. Indeed, we give two  $C^*$ -algebras with isomorphic unitary groups but the algebras themselves are not  $*$ -isomorphic.

## 17:20 – 17:50 A. Badreddine

(Preparatory School in Economics, Business and Management, Algeria)

### *Estimation and Convergence of a Moving Average.*

**Abstract.** We study the method of estimation, and under certain conditions it was the

onvergence of a moving average process.

Let  $\{y_i, -\infty < i < +\infty\}$  be a doubly infinite sequence of identically distributed and dependent random variables .

$\{a_i, -\infty < i < +\infty\}$  be an absolutely summable sequence of real numbers.

Let  $X_n = \sum_{i=-\infty}^{\infty} a_i y_{i+n}, n \geq 1$ , be the moving average process based on the sequence  $\{y_i, -\infty < i < +\infty\}$  .

As usual, we denote  $S_n = \sum_{k=1}^n X_k, n \geq 1$ , the sequence of partial sums.

In my work we discuss the complete convergence of  $(S_n)$  under some suitable

conditions. For example in the  $\phi$ -mixing conditions , and also treat other types of convergence.

**17:50 – 18:20 D. Cardona**

(Universidad de los Andes, Bogotá- Colombia)

*$L^2$ –Estimates for Pseudo-differential Operators on the Torus.*

**Abstract.** In this work we investigate a certain class of pseudo-differential operators defined on the torus. These operators arise from the study of symbols in the classes  $S_{\rho,\delta,\lambda}^m(\mathbb{T}^n \times \mathbb{Z}^n)$ .

## WEDNESDAY

**15:00 – 15:30 C. Cherifa**

(Management and economics preparatory school of Oran, Algeria)

*Commutativity up to a factor for bounded and unbounded operator.*

**15:30 – 16:00 G. De Souza**

(Auburn University, USA)

**16:00 – 16:30 M. Sarih**

(Faculty of sciences Moulay Ismail University, Morocco)

**16:30 – 17:00 D. Israfilov**

(Balikesir University, Turkey)

*Approximation in variable exponent Smirnov classes.*

**17:00 – 17:30 A. Gherbi**

(Science and technics preparatory school of Oran, Algeria)