k-METRIC DIMENSION IN SOME INFINITE GRAPHS AND ULTRAMETRIC SPACES

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Blumental introduced in [1] the metric dimension of a general metric space. This concept has several applications. For example, suppose there is a robot navigating on a network. If it need to calculate its location, then we can put some landmarks in certain vertices, but it is necessary that each vertex can be located by the landmarks. Now, suppose there are two positions which are only distinguished by a single landmark and communication with this landmark is lost. Then, the robot will not be able to locate itself. Thus, in order to improve the accuracy of the detection or the robustness of the system, it may be interesting to have a family of detectors such that every pair of vertices is distinguished by at least k of them. This is the concept of the k-metric dimension which appears in [3] and it is a natural extension of metric dimension. See also [4, 5, 6, 7].

Given a simple and connected graph $G = (V, E)$, a set $S \subseteq V$ is called a *k-metric generator* for G if and only if any pair of different vertices of G is distinguished by at least k elements of S, i.e., for any pair of different vertices $u, v \in V$, there exist at least k vertices $w_1, w_2, \ldots, w_k \in S$ such that

$$d_G(u, w_i) \neq d_G(v, w_i), \text{ for every } i \in \{1, \ldots, k\}.$$

A *k-metric basis* is a k-metric generator of the minimum cardinality in G. Finally, G is said to be a *k-metric dimensional graph* if k is the largest integer such that there exists a k-metric basis for G.

In this sense, we study this problem in the context of some infinite graphs [2] and ultrametric space, which can be interpreted as the end space of a $\mathbb{R}$-tree.

References

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