

## DOMINATION ON HYPERBOLIC GRAPHS

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ABSTRACT. If  $k \geq 1$  and  $G = (V, E)$  is a finite connected graph,  $S \subseteq V$  is said a *distance  $k$ -dominating set* if every vertex  $v \in V$  is within distance  $k$  from some vertex of  $S$ . The *distance  $k$ -domination number*  $\gamma_w^k(G)$  is the minimum cardinality among all distance  $k$ -dominating sets of  $G$ . A set  $S \subseteq V$  is a *total dominating set* if every vertex  $v \in V$  satisfies  $\delta_S(v) \geq 1$  and the *total domination number*, denoted by  $\gamma_t(G)$ , is the minimum cardinality among all total dominating sets of  $G$ . The study of hyperbolic graphs is an interesting topic since the hyperbolicity of any geodesic metric space is equivalent to the hyperbolicity of a graph related to it. In this paper we obtain relationships between the hyperbolicity constant  $\delta(G)$  and some domination parameters of a graph  $G$ . The results in this work are inequalities, such as  $\gamma_w^k(G) \geq 2\delta(G)/(2k + 1)$  and  $\delta(G) \leq \gamma_t(G)/2 + 3$ .

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