Wiener index and average eccentricity for some families of graphs and products

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This talk is concerned with the strong product $G \boxtimes H$ of two graphs, G and H, and bounds on the Wiener index, Hosoya polynomial and the average eccentricity in this product of graphs and other families. We first introduce the distance sequence of a connected graph. It is defined as the sequence of the distances between all unordered pairs of vertices. We prove that the distance sequence of any connected graph of given order and size is dominated by the distance sequence of the so-called path-complete graph. This is the main tool to prove general results as, among others, that, if G is a connected graph of given order and size, then the Wiener index of $G \boxtimes H$, for every fixed connected graph H, and the Hosoya polynomial $W(G, x)$, for every $x \in \mathbb{R}$ with $x \geq 1$, are maximised if G is a path-complete graph. We also investigate the average eccentricity of $G \boxtimes H$. We show that for fixed H, and G chosen from among all connected graphs of given order n, it is maximised if G is a path of the same order. We also determine a graph $G_{n,\delta}$ of order n and minimum degree δ such that for every connected graph G of order n and minimum degree δ, the average eccentricity of $G \boxtimes H$ never exceeds the average eccentricity of $G_{n,\delta} \boxtimes H$ by more than 3.

Keywords.
Wiener index; average distance; average eccentricity; Wiener polynomial; Hosoya polynomial, strong product, distance sequence, distance distribution.

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