# Recent progress on cocyclic matrices

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## 1 Abstract

About twenty-five years ago, Horadam and de Launey introduced the cocyclic development of designs, from which the notion of cocyclic Hadamard matrices developed over a group (of order a multiple of 4) was straightforwardly derived. Furthermore, they showed that the cocyclic framework could provide a structural approach in order to resolve the Hadamard conjecture.

In this talk, we will explain our main progress in this field in the last 10 years, and pose some open problems. Concretely, we will show how we have extended this notion in two directions:

1. Spite of the asymptotic existence results supporting the cocyclic Hadamard conjecture and that many families have revealed to be cocyclic, two of the most prolific constructions of Hadamard matrices have failed to be cocyclic. As an attempt to encompass these families in the cocyclic world, we decided to generalize the theory to cover the wider framework of quasigroups [3, 1].
2. Hadamard matrices can be seen as the solution of the Maximal Determinant Problem (MDP) in a particular case. We have explored to apply the cocyclic approach to the other cases of MDP, considering groups $G$ with orders different from a multiple of 4 [2, 4].

## References

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