La constante de Kemeny de caminos aleatorios con barreras reflejantes

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The computation of the Kemeny's constant is a classical problem in the theory of Markov chains and has multiple applications. The different ways to afford the problem go from Linear Algebra to discrete Potential Theory. The mean first passage time is closely related to other well–known metrics for graphs and Markov chains. First, the Kirchhoff index, also known as the effective graph resistance, is a related metric quantifying the distance between pairs of vertices in an electric network. The relationship between electrical networks and random walks on graphs is well–known. For an arbitrary graph, the Kirchhoff index and the Kemeny constant can be calculated from the eigenvalues of the conductance matrix and the transition matrix, respectively. It is known that there exists an eigenvector called stationary distribution, which gives long-term information of the chain. The short term behavior, can be studied trough the so-called mean first passage time from state i to state j; that will be denoted by $m_{i,j}$. It is the expected number of steps to reach vertex j when starting form vertex i. The mean first passage time matrix can be obtained as the group inverse of the probabilistic Laplacian and some more known date. It is known that the expected time to get any randomly chosen vertex from vertex i is constant and independent of the starting vertex. The common value is called Kemeny constant.

Kemeny's constant can also be written in terms of the eigenvalues for the Probabilistic Laplacian and can be related to Poisson problems state in terms of this Laplacian operator.

Our group has been working during the last decades in this topic, by considering also Dirichlet problems and Neumann problems for more general operators.

When a Markov chain has absorbing states; that is, once the chain enters an absorbing state it can not left (Drunk's walk), the mean first passage time is related with the inverse of some submatrix of the Laplacian, and corresponds to solving Dirichlet problems where the boundary nodes are the absorbing states.

If we want to consider a more safe walk for the Drunk, we need to introduce reflecting barriers. Meaning that if the chain enters the bar with probability one you leave the bar, which represents a reflecting barrier. In terms of operator, this is a Neumann problem; and we investigate which matrix give the mean first passage time. We can also compute M for different families of structured networks; like wheels, paths, fans or ladders.