GENERALIZED CHORDALITY, VERTEX SEPARATORS AND HYPERBOLICITY ON GRAPHS

A. MARTÍNEZ-PÉREZ

Facultad de Ciencias Sociales, Universidad de Castilla-La Mancha
Avda. Real Fbrica de Seda, s/n. 45600 Talavera de la Reina, Toledo, Spain
e-mail: alvaro.martinezperez@uclm.es

Let G be a graph with the usual shortest-path metric. A graph is δ-hyperbolic if for every geodesic triangle T, any side of T is contained in a δ-neighborhood of the union of the other two sides. A graph is chordal if every induced cycle has at most three edges. A vertex separator set in a graph is a set of vertices that disconnects two vertices.

Being hyperbolic is an important property in metric spaces and being chordal is also a deeply studied property on graphs. In [3], the authors prove that chordal graphs are hyperbolic giving an upper bound for the hyperbolicity constant. In [8], Wu and Zhang extend this result for a generalized version of chordality. They prove that k-chordal graphs are hyperbolic where a graph is k-chordal if every induced cycle has at most k edges. In [1], the authors define the more general properties of being (k,m)-edge-chordal and $(k,\frac{k}{2})$-path-chordal and prove that every (k,m)-edge-chordal graph is hyperbolic and that every hyperbolic graph is $(k,\frac{k}{2})$-path-chordal. In [6], we continue this work and define being ε-densely (k,m)-path-chordal and ε-densely k-path-chordal relating these properties with hyperbolicity and giving a characterization of being hyperbolic in terms of chordality.

G. A. Dirac proved in [4] that a graph is chordal if and only every minimal vertex separator is complete. See also [5, 7] and [2] and the references therein.

In this work we study the relation between vertex separator sets, the generalized chordality properties mentioned above and the hyperbolicity of the graph. We also give a characterization of being quasi-isometric to a tree in terms of chordality and prove that this condition also characterizes being hyperbolic, when restricted to triangles, and having stable geodesics, when restricted to bigons.

References

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