The exact value of 3-color off-diagonal generalized Schur numbers $S(2, 2, k)$

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Abstract

For integers $a \leq b$, we shall denote $[a, b]$ the *integer interval* consisting of all $t \in \mathbb{N}_+ = \{1, 2, \dots\}$ such that $a \leq t \leq b$. A function

$$\Delta : [1, N] \longrightarrow \{d_1, \dots, d_r\},$$

where $d_1, \dots, d_r \in \mathbb{N}_+$ represent different colors, is a r-coloring of the interval $[1, N]$.

Given a r-coloring Δ and the equation $E_k : x_1 + \cdots + x_k = x_{k+1}$ in $k+1$ variables, then we say that a solution $x_1, \dots, x_k, x_{k+1}$ to the equation E_k is monochromatic if and only if $\Delta(x_1) = \Delta(x_2) = \cdots = \Delta(x_{k+1})$.

For integers r and k_i, with $r \geq 2$ and $k_i \geq 2$ for $i = 1, \dots, r$, the *r-color off-diagonal generalized Schur number* denoted by $S(k_1, k_2, \dots, k_r)$ is defined as the least integer M such that any r-coloring of the integer interval $[1, M]$ must admit a j-colored solution to equation $E_{k_j} : x_1 + x_2 + \ldots + x_{k_j} = x_{k_{j+1}}$ for some j with $1 \leq j \leq r$.

In this work, we determine the exact value of $S(2, 2, k)$ with $k \geq 2$.

Keywords: Schur numbers; sum-free sets; off-diagonal Schur numbers.

MSC 2010: 05C55; 05D10; 05-04; 05A17.