Lower bounds and exact values on the weak Schur numbers

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For integers k, n with k, $n \geq 1$, the *n-color weak Schur number* $WS_k(n)$ is defined as the least integer N, such that for every n-coloring of the integer interval $[1, N]$, there exists a monochromatic solution $x_1, \ldots, x_k, x_{k+1}$ in that interval to the equation:

$$x_1 + x_2 + \cdots + x_k = x_{k+1},$$

with $x_i \neq x_j$, when $i \neq j$.

In this work, we obtain the exact values of $WS_6(2) = 166$, $WS_7(2) = 253$, $WS_3(3) = 94$ and $WS_4(3) = 259$ and we show lower bounds on *n-color weak Schur number* $WS_k(n)$ for $n = 2, 3, 4$.

We determine the exact values finding upper bounds which coincide with the general lower bounds translating the problem into a Boolean satisfiability problem, which can be handled by a SAT solver.

Keywords: Schur numbers; sum-free sets; weak Schur numbers; weakly sum-free sets; n-coloring.