The continuous mean distance of a graph

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We study the concept of the *continuous mean distance* of a weighted graph. The *mean distance* of a connected unweighted graph is often defined as the arithmetic mean of all nonzero distances between vertices, where the distance between vertices u and v is the length of a shortest path connecting u and v, and the sum is taken over all unordered pairs of vertices in the graph. This graph parameter provides a natural measure of the compactness of the graph, and has been intensively studied, both for unweighted and weighted graphs.

In this work we study the *continuous* analog of the mean distance, defined for connected weighted graphs with non-negative weights satisfying the triangle inequality. Conceptually, the continuous mean distance is the mean of the distances between all pairs of *points* on an edge of the graph, where a *point* p on an edge $e = uv$ can be expressed as $p = \lambda_p v + (1 - \lambda_p)u$ for some $\lambda_p \in [0, 1]$.

Despite being a very natural generalization, to the best of our knowledge this concept has been barely studied, the only exception being the preliminary work by Doyle and Graver [1], where the authors define the mean distance of a *shape* (or type of arrangement) through a limiting process, and compute its value for a few shapes.

In this talk we will introduce the continuous mean distance, some of its computational aspects, and we will discuss its relation with its well-known discrete counterpart.

References

[1] J. K. Doyle and J. E. Graver. Mean distance for shapes. *J. Graph Theory*, 6(4):453–471, 1982.