Mathematical Art in Japan

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What does it take to appreciate a musical piece? Most people must hear the music played. If it is played by a full orchestra in a grand concert hall, then perhaps it will be memorable. What does it take to appreciate a recipe? The dish must be prepared and tasted. If it is presented like some work of art and savored in the ambiance of a great restaurant, then maybe it will be remembered. Jin Akiyama, Professor of Mathematics and Director of the Research Institute of Educational Development at Tokai University in Japan, holds that a similar statement can be made about mathematics: memorable mathematics engages the senses. This principle has guided him and his colleagues to produce the exhibit *Mathematical Art,* consisting of physical models that vividly demonstrate mathematical concepts, formulas and theorems. The attraction of the exhibit is that viewers are encouraged to manipulate the models, to experiment, and experience mathematics.

Since 1998, *Mathematical Art* has traveled to a number of cities in Japan and has attracted record-breaking numbers of viewers. The exhibit in Ueno (Tokyo) lasted for three weeks and drew approximately 49,000 viewers, while the Hiroshima exhibit ran for a month and was visited by approximately 66,000 viewers. The exhibit was also staged in Asahikawa, Shonan and Kumamoto and in neighboring Asian cities, Seoul, Korea and Manila, Philippines. It was shown for two days in conjunction with ICME 9 in Makuhari in August 2000 and attracted 16,900 viewers. What was most striking to the participants at ICME 9 was that there were no school organized groups of children there, instead there were family groups, parents enjoying the exhibit with their children.

Mathematical Art was also shown in Shizuoka, in Aoyama (Tokyo), and in Bond University in Australia. Some models from *Mathematical Art* are now on permanent exhibit in the **Shizuoka Science Museum**. An expanded version of *Mathematical Art* is also on permanent exhibit in **Ohotsuku Mathematics Wonderland** in Northern Japan, a venue developed under the initiative of Akiyama in coordination with the City Council of Ohotsuku.

[pictures taken at various exhibits]

This year, 2004, models from *Mathematical Art* will form part of the UNESCO exhibit to promote mathematics. The exhibit venues include Stockholm, Sweden, Copenhagen, Denmark during ICME 10, Orleans and Paris in France.

Although Japanese students always perform very well in international mathematics competitions and examinations, a survey associated with TIMSS (The Third International Mathematics and Science Study) indicates that 40% of all Junior High School students in Japan do not like mathematics. Subsequent interviews indicate that they are overwhelmed by so much content – introductory calculus, probability and statistics, matrix theory, and they do not appreciate the many formulas they have to memorize.

Jin Akiyama observes that mathematics, as it is traditionally taught in most Japanese classrooms, fails to elicit wonder and fails to foster creativity. The models in the *Mathematical Art* exhibit aim to restore some of this wonderment. Each vividly demonstrates the power and beauty of mathematics. The models were crafted by mathematicians, sculptors and industrial designers from the faculty and staff of Tokai University.

The Exhibit

Mathematical Art includes more than 200 models and continues to grow. The composition of each exhibit varies somewhat but includes at least a hundred models at a time. The models range from palm-size to 8-feet tall. The heaviest weighs 500 kgs. Several are run by motors. Some cost as much as a million yen. The large models were fabricated at the Hokkaido Tokai University School of Art and Design.

Most models demonstrate well-known mathematical formulas and theorems. Some are devised from models described in books and magazine articles. Others illustrate recent results obtained by Jin Akiyama and his associates.

Favorite Models in the Exhibit

Viewer surveys point to some favorite models. Two rotary models demonstrating the Pythagorean Theorem always manage to attract viewers who eventually ask to have their picture taken beside one or the other. One model has three thin containers with square cross sections mounted on a circular base. The dimensions of the squares are determined by the Pythagorean Theorem. Initially, each of the smaller containers is filled with pieces of colored plastic. As the circular base rotates, the pieces of plastic from the smaller containers fall into the larger container and fill it up exactly. The other has containers of the same size as the first model and the same set-up except that the two smaller containers are filled with colored water. As the circular base rotates, the water flows into the larger container and fills it exactly by the time that the two smaller ones empty completely.

[pictures of rotary models demonstrating the Pythagorean Theorem]

The *cradle pinball device* consists of a board mounted on a curved base which allows the board to tilt in two possible directions. The board has a reservoir on one end

for holding balls, has pins placed at equal intervals along horizontal lines going towards the opposite end of the board, and compartments at the opposite end.

A gate holds the balls in place in the reservoir. When the device is tilted in the direction of the compartments and the gate is opened, the balls roll down. The pins in succeeding horizontal lines are placed at angles so that a ball, rebounding from a pin as it rolls down, has equal chances of falling to the left or right of a lower pin.

The device can effectively demonstrate several probability distributions as follows. Start by simply tilting the device in the direction of the reservoir and closing the gate after all the balls are collected.

To demonstrate the *binomial distribution*, tilt the device in the direction of the compartments, open the gate, and let the balls roll down. With unfailing regularity, the balls will fall into the compartments to form a configuration which is unimodal. How does this happen? Consider the triangular configuration of pins whose apex is just under the gate of the reservoir.

There is only one path to the apex. After the ball rebounds from the apex, it has equal chances of falling to the left or right of the apex i.e., the probability would be in the proportion 1:1. Hence the probabilities of falling to the left of both of the two lower pins, between them, or to the right of both would be in the proportion 1:2:1. Continuing the process, the probability of a ball passing between different pins in a row, or to the left or right of the entire row is in the following proportions.

1
1 ● 1
1 ● 2 ● 1
1 ● 3 ● 3 ● 1
1 ● 4 ● 6 ● 4 ● 1

In general, the distribution of probabilities in the n^{th} row is proportional to the coefficients in the expansion of $(a+b)^n$.

Other distributions such as the *geometric distribution*, the *Poisson distribution*, the *hypergeometric distribution*, and the *compound normal distribution* can be obtained by placing bars or triangular frames over the pins on the board.

[pictures of the cradle pinball device]

An ingenious model obtains the greatest common factor (gcf) and least common multiple (lcm) of two natural numbers automatically. To keep the device portable, the natural numbers are limited to those whose prime factors are 2, 3, and 5 but the principles used in the construction of the device can be extended to include larger factors.

Their prime factorizations of two numbers *a* and *b* are first determined and then represented by balls of graduated sizes, based on the sizes of the primes, and marked accordingly. The device has two funnels. The balls in the representation of *a* are placed in one funnel and the balls in the representation of *b* in the other. Within the device are sorting and balancing mechanisms which make use of the sizes and weights of the balls. The process sends the balls of the same size from *a* and *b* into adjacent compartments. Of the two adjacent compartments, the one with fewer balls rises while the one with more balls falls. To take care of cases where there are an equal number of balls in each compartment, a small weight has been placed to favor the right compartment; this weight is too small to affect the general case.

The numbers appearing on the balls in the higher level compartments are multiplied to obtain the gcf. The same is done with the numbers appearing on the balls in the lower compartments to obtain the lcm.

An unusual set of models consists of measuring devices without gradations but which can measure any integral amount of liquid up to their total capacity. Liquid is allowed to be scooped up only once from the original container.

One such device is a rectangular parallelepiped which has a capacity of 6 liters. It can measure various amounts of liquid by simply being tilted appropriately. For example, to measure one liter, tilt the device so that the liquid takes the shape of a triangular pyramid. Since its base is half the base of the device, the amount of liquid is

$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ of the capacity of the cup or one liter. Three liters can be measured by tilting the device so that the liquid forms a triangular prism covering half of the volume of the device.

To pour 4 liters, first scoop 6 liters, then tilt to pour 3 liters into a second container, then tilt to pour back a liter into the original container; finally, pour the remaining liter into the second container.

Other devices have a capacity of 10, 20 and 114 liters.

[picture of rectangular parallelepiped and other measuring devices]

Another set of models extends the principle operating in the kaleidoscope to create fascinating images using a few pieces of plastic and some reflecting mirrors. Images of the five platonic solids (cube, tetrahedron, octahedron, dodecahedron, and icosahedron) can be obtained in this manner.

The arrangement of mirrors in each model is called a *mirror cone*. To create the image of an icosahedron, a mirror cone consisting of three isosceles triangles of the dimensions shown in the table is used. A plastic equilateral triangle is set inside the cone. The icosahedron can be seen by looking into the mirror cone.

Images of the other platonic solids can be obtained using the mirror cones specified in the table below. One face of the desired polyhedron is set inside the mirror cone.

Solid	**Mirror Cone**	**Dimensions of the Triangular mirrors**
Cube	Quadrangular	70°36' 54°44' 54°44'
Tetrahedron	Triangular	109°28' 35°16' 35°16'
Octahedron	Triangular	90° 45° 45°
Dodecahedron	Pentagonal	41°48' 69°06' 69°06'
Icosahedron	Triangular	63°26' 58°17' 58°17'

[pictures of the images of the platonic solid]

The models for reversible solids are startling constructions. One parallelepiped is partitioned specifically so that it can be folded outward and transformed into a truncated octahedron. Another, parallelepiped can be transformed into a rhombic dodecahedron. The exhibit also includes a truncated octahedron which can be transformed into a solid congruent to itself.

The Man Behind the Exhibit

The driving force behind *Mathematical Art* is Jin Akiyama. He is known in the mathematics community as a graph theorist. At the University of Michigan, he worked with Frank Harary, a taskmaster who challenged him with the requirement "another day another paper." He subsequently obtained a Doctor of Science degree from Science University of Tokyo on the basis of his published papers in graph theory.

Akiyama is Editor-in-Chief of the journal *Graphs and Combinatorics*. He organized the Japan conferences in graph theory and combinatorics held in 1986 and 1990. These conferences attracted an international cast luminaries of graph theory and propelled Japanese mathematicians to the forefront of research this rich area. Currently, he is a member of the National Curriculum Committee which recommends policies for basic education in Japan.

Long regarded as a charismatic teacher, Akiyama is very much in demand in the Japanese lecture circuit. On any week, he travels at least twice a week from Tokyo to some region in Japan to address audiences of no less than 500 at a time. His lectures always include striking demonstrations of the power and beauty of mathematics. In the audience are concerned parents who worry about their children's performance in mathematics class and who ask his advice, or high school teachers who are searching for innovative ways of teaching mathematics, or students who later crowd around him to ask for his autograph, or even retirees who are just eager to continue learning.

[pictures of Akiyama lecturing and large audience]

It did not take long for the television network NHK (Nihon Housou Kyokai, literally translated as Japan Broadcast Association) to offer him a series of his own shows to teach mathematics on television in 1991. He has completed the fourteenth series. He works with research assistants, writers, and a television production crew. Each series takes six months of preparation and every 30-minute segment takes about six hours to tape. Most of the models shown in the *Mathematical Art* exhibit have been featured in these series. The series have also led to the production of associated videos and workbooks.

Always on the lookout for sources of new and intriguing mathematics, Akiyama has recently taken an interest in computational and discrete geometry, an area which covers some common ground with graph theory. Old friends from the graph theory community like Vasek Chvatal, David Avis, Janos Pach and Jorge Urrutia, who also publish in this area, are sources of encouragement. He organized the Japan Conference in Discrete and Computational Geometry (JCDCG 1998). Succeeding international conferences in this series were held in 2000 and 2002. Proceedings of these conferences were published as part of Springer-Verlag's Lecture Notes in Computer Science (LNCS). Another conference is planned for October 2004. Since Akiyama integrates all of his interests, models from discrete and computational geometry have found their way into the *Mathematical Art* exhibit.

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