1 Angel Ballesteros

Lorentzian Poisson homogeneous spaces

Angel Ballesteros[1], Iván Gutiérrez Sagredo[1], Francisco J. Herranz[1]
[1] Departamento de Física, Universidad de Burgos, E-09001 Burgos, Spain
e-mail: angelb@ubu.es, igsagredo@ubu.es, fjherranz@ubu.es

Abstract

Poisson homogeneous spaces (M, π) of a Poisson–Lie group (G, Π) are introduced, where M is given by $M = G/H$ and H is a given isotropy subgroup $H \subset G$. If $(\mathfrak{g}, \delta)$ is the Lie bialgebra associated to (G, Π), the characterization of Poisson homogeneous spaces (hereafter PHS) under (G, Π) is given in terms of the coisotropy condition for the cocommutator δ, which means that $\delta(\mathfrak{h}) \subset \mathfrak{h} \wedge \mathfrak{g}$, where $\mathfrak{h} = \mathrm{Lie}(H)$. The particular case when $\delta(\mathfrak{h}) \subset \mathfrak{h} \wedge \mathfrak{h}$ identifies the Poisson subgroup cases, which are less numerous but can be more easily promoted to quantum homogeneous spaces (see [1, 2, 3] and references therein).

In this contribution we present the explicit construction of some PHS on Lorentzian spacetimes. In particular, all PHS for the AdS group in (1+1) dimensions are studied. In (2+1) dimensions, Minkowski PHS coming from Drinfel'd double structures of the (2+1) Poincaré Lie algebra are systematically constructed [4]. Finally, a PHS for the (3+1) dimensional AdS group with respect to the so-called κ-Poisson-Lie structure is explicitly given, and its Minkowskian limit is obtain in the vanishing cosmological constant limit [5].

Finally, we will comment on the physical relevance of Lorentzian PHS as semiclassical counterparts of quantum homogeneous spaces. In this context, the Poisson homogeneous structure π on the classical Lorentzian spacetime M would provide a semiclassical signature of the noncommutativity between the spacetime coordinates that would be one of the possible footprints of Quantum Gravity effects at the Planck scale.

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2 Anna Maria Candela

Gravitational Waves: some Geometric Properties

Anna Maria Candela[a]

[a] Università degli Studi di Bari Aldo Moro. Bari, Italy
E-mail: annamaria.candela@uniba.it

Abstract

Since 1974, many times indirect proofs of the existence of gravitational waves have been given but only in September 2015 physicists gave the official announcement that gravitational waves exist.

Anyway, from a mathematical point of view, they have always been “real objects” and their geometric properties such as geodesic connectedness and geodesic completeness have been investigated.

These results are in joint works with Rossella Bartolo, José Luis Flores, Alfonso Romero and Miguel Sánchez.

3 Ildefonso Castro

Prescribing curvature for curves in Lorentz-Minkowski plane

Ildefonso Castro[a] Ildefonso Castro-Infantes[b] and Jesús Castro-Infantes[c]

[a] Departamento de Matemáticas, Universidad de Jaén, 23071 Jaén, Spain
E-mail: icastro@ujaen.es

[b] Departamento de Geometría y Topología, Universidad de Granada, 18071 Granada, Spain
E-mail: icastroinfantes@ugr.es
[c] Departamento de Geometría y Topología, Universidad de Granada, 18071 Granada, Spain
E-mail: jcastroinfantes@ugr.es

Abstract

Motivated by a problem proposed by David A. Singer in 1999 and by the classical Euler elastic curves, the aim of this talk is to study spacelike and timelike curves in Lorentz-Minkowski plane $\mathbb{L}^2$ with prescribed curvature. Specifically, we try to determine those curves $\gamma = (x, y)$ in $\mathbb{L}^2$ whose curvature κ depends on some given function $\kappa = \kappa(x, y)$. Concretely, we study (cf. [1] and [2]) spacelike and timelike curves whose curvature is expressed in terms of the Lorentzian pseudodistance to fixed geodesics (of different causality) or to a fixed point.

From a geometric-analytic point of view, we deal with the following problem: Given a unit-speed parametrization of a spacelike or timelike curve $\gamma = (x, y)$ in $\mathbb{L}^2 := (\mathbb{R}^2, g = -dx^2 + dy^2)$, we prescribe the curvature with some of the following extrinsic conditions:

(i) $\kappa = \kappa(y)$ or $\kappa = \kappa(x)$,

(ii) $\kappa = \kappa(v)$, where $v := y - x$,

(iii) $\kappa = \kappa(\rho)$, where $\rho := \sqrt{|g(\gamma, \gamma)|} = \sqrt{|-x^2 + y^2|} \geq 0$.

In this way, we get a complete description of the spacelike and timelike elastic curves in $\mathbb{L}^2$ and provide the Lorentzian versions of catenaries and grim-reaper curves. We also find out several new families of Lorentzian spacelike and timelike spirals. In addition, we provide uniqueness results for the generatrix curve of the Enneper's surface of second kind and for Lorentzian versions of some well known curves in the Euclidean setting, like the Bernoulli lemniscate, the cardioid, the sinusoidal spirals and some non-degenerate conics.

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4 Rui Loja

Genus Integration, Abelianization and Extended Monodromy

Rui Loja Fernandes[a]

[a] Department of Mathematics, University of Illinois at Urbana-Champaign, USA
E-mail: ruiloja@illinois.edu

Abstract

Given a Lie algebroid we discuss the existence of an abelian integration of its abelianization. We show that the obstructions are given by the so-called extended monodromy groups introduced recently. We also show that the abelianization can be obtained by a path-space construction, similar to the Weinstein groupoid, but where the underlying homotopies are now supported in surfaces with arbitrary genus, i.e., instead of A-homotopies one considers A-homologies. This talk is based on joint work with Ivan Contreras (Amherst College).

5 Juan Manuel Pérez-Pardo

Quantum Control on the Boundary

Juan Manuel Pérez-Pardo[a]

[a] Universidad Carlos III de Madrid
E-mail: jmppardo@math.uc3m.es

Abstract

Schroedinger equation is a linear evolution equation. The problem of controlling a finite dimensional quantum system is therefore a well understood problem where one can apply the classical theory of control. However, applying such ideas to the infinite dimensional setting is far from being straightforward. For instance, one immediately encounters difficulties with the definition of the dynamical Lie algebra due to the appearance of unbounded operators.

In spite of the technical difficulties, the latter bring also new and interesting possibilities to the theory of quantum control. We will introduce notions of controllability suited for the infinite-dimensional situation and discuss how one can control the state of a system by means of changing its boundary conditions.

6 Patricia Santos

Jacobi multipliers and nonholonomic Lagrangian systems

José F. Cariñena[a] and Patrícia Santos[b]

[a] Department of Theoretical Physics, University of Zaragoza, Spain
E-mail: jfc@unizar.es

[b] CMUC, Department of Mathematics, University of Coimbra
and Coimbra Polytechnic - ISEC, Coimbra, Portugal
E-mail: patricia@isec.pt

Abstract

We extended our previous work [1] that establish the relation between Jacobi multipliers (see e.g. [2] and references therein) and Hamel's formalism [3], to more general cases where the configuration space is not the Euclidian space. Specifically, several results and examples of nonholonomic Lagrangian systems on Riemannian manifolds are given.

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7 Alfonso G. Tortorella

Contact Dual Pairs

ADARA BLAGA[a], MARIA AMELIA SALAZAR[b], ALFONSO GIUSEPPE TORTORELLA[c] AND CORNELIA VIZMAN[d]

[a] Department of Mathematics, West University of Timisoara, Romania.
E-mail: adarablaga@yahoo.com

[b] Departamento de Matemática Aplicada, Universidade Federal Fluminense, Niterói, RJ, Brazil.
E-mail: mariasalazar@id.uff.br

[c] Department of Mathematics, KU Leuven, Belgium
E-mail: alfonsogiuseppe.tortorella@kuleuven.be

[d] Department of Mathematics, West University of Timisoara, Romania.
E-mail: cornelia.vizman@e-uvt.ro

Abstract

In this work [1] we investigate the notions of duality and dual pairs in Jacobi geometry [2]. We introduce a contact dual pair as a pair of Jacobi bundle maps defined on the same (generically non-coorientable) contact manifold and satisfying a certain orthogonality condition. The standard example is formed by the source and the target maps of a contact groupoid. Among various properties, we investigate the relation existing between symplectic dual pairs [3] and contact dual pairs via symplectization. Our main result is the proof of the characteristic leaf correspondence theorem for contact dual pairs. Indeed there is a one-to-one correspondence between the characteristic (symplectic or contact) leaves of the two Jacobi manifolds forming the legs of a contact dual pair with connected fibers. Finally we apply these results to the context of reduction theory. Indeed we prove that any free and proper contact groupoid action gives rise to the contact dual pair formed by the associated moment map and the projection on the orbit space. Consequently the characteristic leaf correspondence yields a new insight into the contact reduction method first introduced by Zambon and Zhu [4].

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8 Raquel Villacampa

Solutions of the Laplacian flow and coflow of a locally conformal parallel G_2-structure

Victor Manero[a], Antonio Otal[b] and Raquel Villacampa[b]

[a] Facultad de Ciencias Humanas y de la Educación. Huesca
E-mail: vmanero@unizar.es

[b] Centro Universitario de la Defensa, Academia General Militar, 50009, Zaragoza
E-mail: aotal@unizar.es
E-mail: raquelvg@unizar.es

Abstract

The development of flows in Riemannian geometry has been mainly motivated by the study of the Ricci flow. However, the same techniques are also useful in the study of flows involving other geometrical structures, like for example, the Kähler Ricci flow. Concerning flows on G_2-manifolds, for any closed G_2-structure σ_0 on a manifold M, Bryant (Proceedings of Gökova Geometry-Topology Conference 2005, 2006) introduced a natural flow, the so-called *Laplacian flow*, given by

$$\begin{cases} \frac{d}{dt}\sigma(t) = \Delta_t \sigma(t), \\ \sigma(0) = \sigma_0, \quad d\sigma(t) = 0, \end{cases}$$

where Δ_t is the Hodge Laplacian operator of the metric determined by $\sigma(t)$. The short time existence and uniqueness of solution for the Laplacian flow of any closed G_2-structure, on a compact manifold M, has been proved by Bryant and Xu in the unpublished paper arxiv:1101.2004[math.DG].

Karigiannis, McKay and Tsui (Diff. Geom. Appl. 2012) introduced the *Laplacian coflow.* In this case the initial G_2-form is claimed to be coclosed, i.e. $d * \sigma_0 = 0$. Up to now, short time existence of solution of the coflow is not known. Assuming short time existence and uniqueness of solution, the authors show that the condition of the initial G_2-form σ_0 to be coclosed (equiv. ψ_0 closed) is preserved along the flow.

Here we are concerned with studying the Laplacian flow, resp. coflow, of an LCP G_2-structure on a manifold M defined as:

$$\begin{cases} \dfrac{d}{dt}\sigma(t) = \Delta_t \sigma(t), \\ \sigma(0) = \sigma_0, \\ d\sigma(t) = 3\,\tau(t) \wedge \sigma(t), \\ d *_t \sigma(t) = 4\,\tau(t) \wedge *_t \sigma(t). \end{cases} \qquad \begin{cases} \dfrac{d}{dt}\psi(t) = -\Delta_t \psi(t), \\ \psi(0) = \psi_0, \\ d\psi(t) = 4\,\tau(t) \wedge \psi(t), \\ d *_t \psi(t) = 3\,\tau(t) \wedge *_t \psi(t). \end{cases}$$

The first examples of long time solutions of these flows are given. Our examples are one-parameter families of Locally Conformal Parallel G_2-structures on solvable Lie groups. We start finding solutions for the Laplacian flow and the found solutions are used to construct long time solutions to the Laplacian coflow starting from a Locally Conformal Parallel structure. These results can be found in the preprint available in arxiv:1711.08644[math.DG].