First Joint Meeting between the RSME and the AMS

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Abstracts

Session 01

Affine Algebraic Geometry

Organizers:

Jaime Gutiérrez (Universidad de Cantabria)
Vladimir Shpilrain (The City College of New York)
Jie-Tai Yu (The University of Hong-Kong)

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Jacobian quotients of polynomial mappings

E. Artal Bartolo (University of Zaragoza)

Let $\phi := (f, g) : \mathbb{C}^2 \to \mathbb{C}^2$ where f and g are polynomial maps. A relationship is established between the following two objects: on the one hand, the Newton polygon of the union of the discriminant curve of ϕ and its non-properness locus, and on the other, the topological type of the link at infinity of the affine curves $f^{-1}(0)$ and $g^{-1}(0)$. This is a joint work with P. Cassou-Noguès and H. Maugendre.

Purely inseparable k-forms of affine algebraic curves

T. Asanuma (Toyama University)

Let k be a field of characteristic $p > 0$ and let k' be a purely inseparable closure of k. Given a commutative k-algebra B, a k-algebra A (resp. an algebraic scheme $\operatorname{Spec} A$) is called a purely inseparable k-form of B (resp. $\operatorname{Spec} B$), if $A \otimes k' \cong B \otimes k'$.

Theorem Let B be a one dimensional affine k-domain such that $B \otimes k'$ is a regular domain with the differential $B \otimes k'$-module over k' is free. Suppose $p > 2$. Then the following two conditions are equivalent:
(1) A is a purely inseparable k-form of B.
(2) There exists an integer $e \geq 0$ such that

$$A \cong k[B^{p^e}, t^2u, tu^q] \ (q = (p^e + 1)/2),$$

where $t, u \in k^{1/p^e}[B](\subset B^{1/p^e})$ satisfy an equation

$$td(u) + 2ud(t) = 1$$

for a k^{1/p^e}-derivation d of $k^{1/p^e}[B]$.

As an application we obtain a structure theorem of all k-forms of affine plane curves $W = \operatorname{Spec} k[x, y]/(f(x, y))$, when $p > 2$ and W is geometrically regular and geometrically integral. In particular if the group of k'-automorphisms of $k'[x, y]/(f(x, y))$ is finite, then there exist purely inseparable k-forms V of W such that $V \ncong W$.

Birational maps from $\mathbb{C}^2$ to $\mathbb{C}^2$

P. Cassou-Noguès (Université Bordeaux I)

I will speak about a joint work (in progress) with P. Russell.

Considering birational maps from $\mathbb{C}^2$ to $\mathbb{C}^2$, can be seen as considering

$$\begin{array}{ccc} Z & \longrightarrow & X = \mathbb{C}^2 \\ \uparrow & \nearrow & \\ Y = \mathbb{C}^2 & & \end{array}$$

where $Z = \mathbb{F}_l \setminus S$ with $S^2 = k + 1$, $Y = Z \setminus C$. Then the problem of studying birational maps from $\mathbb{C}^2$ to $\mathbb{C}^2$, breaks in two parts

i) describing the morphisms $Z \longrightarrow X$

ii) describing all $Y \subset Z$, $Y = Z \setminus C$ and $Y \simeq \mathbb{C}^2$.

I will explain that for some classes of birational maps.

Affine surfaces with trivial Makar-Limanov invariant

D. Daigle (University of Ottawa)

I will present joint work of Peter Russell and myself on the class of normal affine surfaces S satisfying:

$$\mathrm{ML}(S) \text{ is trivial and } \mathrm{Pic}(S') \text{ is finite,}$$

where S' is the smooth locus of S. Our results include a complete classification of these surfaces and of the G_a-actions on them. We also realize these surfaces as open subsets of weighted projective planes. This classification is an elaborate corollary to our previous work on affine rulings of weighted projective planes.

Embeddings of Danielewski Surfaces

G. Freudenburg* (University of Southern Indiana)

L. Moser-Jauslin (Université de Bourgogne)

A *Danielewski surface* S is any surface algebraically isomorphic to a surface in $\mathbf{C}^3$ defined by a polynomial of the form $P = x^n z - p(y)$. In addition, S is called *special* iff its Makar-Limanov invariant equals $\mathbf{C}$. Note that a plane is a special Danieleski surface.

For every non-special Danielewski surface S, we produce a polynomial P' such that: (1) the fibers defined by $P - a$ and $P' - a$ are holomorphically isomorphic ($a \in \mathbf{C}$); (2) the non-zero fibers of P and P' are algebraically non-isomorphic; (3) the zero fibers of P and P' are algebraically isomorphic. This implies that the two embeddings of S defined by these zero fibers are algebraically non-equivalent.

In the case of certain special (non-planar) Danielewski surfaces, we produce P' such that the non-zero fibers are algebraically isomorphic, but the zero fibers are not. This implies that the embeddings of S defined by the non-zero fibers are not equivalent.

Our work is motivated by methods of Masuda and Petrie, who considered non-linearizable algebraic actions of $O(2)$ on $\mathbf{C}^4$ introduced by Schwarz. We also draw on recent results of Makar-Limanov and Daigle concerning Danielewski surfaces.

Some new examples of 4-dimensional algebraic exotic structures

M. González Villa (Universidad Complutense de Madrid)

Kaliman and Makar Limanov proved in [4] that the Koras–Russell threefolds were algebraic exotic structures. This result led to the linearization of C^*–actions on C^3, [2]. We present new examples of 4–dimensional exotic structures that somehow generalize the exotic Koras–Russell threefolds. The examples enlighten the relation between the Derksen and Makar–Limanov invariants and prove a conjecture of Artal–Bartolo, Luengo and Melle.

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Computing Unirational Fields of arbitrary transcendence degree

J. Gutierrez* (Universidad de Cantabria)

R. Rubio (Universidad de Cantabria)

D. Sevilla (Universidad de Cantabria)

In this talk we discuss the following computational problem: given an extension $\mathbb{K}(f_1, \dots, f_m) \subset \mathbb{K}(x_1, \dots, x_n)$, to compute an intermediate field $\mathbb{F}$ which is algebraic over $\mathbb{K}(f_1, \dots, f_m)$, where $\mathbb{K}$ is a field of characteristic zero. This question has been considered by various authors when the transcendence degree of $\mathbb{K}(f_1, \dots, f_n)$ is equal to 1 and when it is equal to n. They presented methods to find such intermediate fields, and if $n = 1$ there are algorithms in polynomial time.

Our approach requires to compute Groebner Basis, factorization in algebraic extensions and a generalization of a result due to J. Schicho relating multivariate factorization to decomposition problems.

We also study the generalization of these problems to the case where the field $\mathbb{K}(x_1, \dots, x_n)$ is replaced by the field of fractions of $\mathbb{K}[x_1, \dots, x_n]/J$ for some prime ideal J and also for positive characteristic fields.

Inverse Limits of Polynomial Rings

T. Kambayashi (Tokyo Denki University, Saitama)

Let $A = \lim_{\leftarrow,i} A_i$ be a pro-affine algebra over a field K (see Kambayashi [J. of Algebra, **185**(1996),481–501; Osaka J. of Math., to appear]). Assume all A_i's to be reduced, and look at a pro-affine algebra $\tilde{A} := \lim_{\leftarrow,i} A_i[T_i]$ of polynomial algebras. We have studied the question as to whether $\tilde{A}$ is a polynomial A-algebra $A[Y]$ for a suitably chosen "variable" $Y \in \tilde{A}$, and have gotten two answers:

Theorem 1. Let $P = (P_0 \leftarrow P_1 \leftarrow \cdots \leftarrow P_i \leftarrow \cdots) \subset A$ be any open prime and let $S_i := A_i - P_i$ $(\forall i)$. Then, $\tilde{A}$ is locally a polynomial algebra above P, *i.e.*

$$\lim_{\leftarrow}((S_i^{-1}A_i)[T_i]) \simeq (\lim_{\leftarrow}(S_i^{-1}A_i))[Y] = A_P[Y].$$

Next, consider the inverse system $\mathbf{U} := (U_i : i \geq 0)$ of units U_i of A_i. A is said to be *uniformly stable for units* if $\mathbf{U}$ satisfies the Mittag-Leffler condition uniformly at every level i.

Theorem 2. Assume A to be uniformly stable for units. Then, every $\tilde{A} = \lim_{\leftarrow,\, i} A_i[T_i]$ is isomorphic to an $A[Y]$.

The explicit factorization of the Cremona transformation which is an extension of the Nagata automorphism into elementary links

T. Kishimoto (Max-Planck-Institut fur Mathematik)

It is well known that every automorphism on the affine plane C^2 is tame. In order to see this fact from a point of view of algebro-geometric method, it is important to factorize Cremona transformations on P^2 which are obtained as extensions of automorphisms on C^2 as performed, for example, in Miyanishi's book: "Lectures on curves on rational and unirational surfaces". Meanwhile, the structure of automorphism groups on the affine spaces for higher dimensional case is not understood at the present time completely. For instance, we do not know whether the famous Nagata automorphism on the affine 3-space C^3 is tame or not. (I heared that Shestakov and Umirbaev seem to show that the Nagata automorphism is not tame by algebraic method. But I can not obtain their preprint.) In my opinion, it is an important step to observe the Cremona transformation on P^3 obtained as an extension of the Nagata automorphism in order to decide whether it is tame or not by an algebro-geometric method as in the two-dimensional case. In my talk, we shall factorize this Cremona transformation on P3 explicitly into a finite number of elementary links according to the Sarkisov Program.

Tame and wild coordinates of R[x,y]

C. M. Lam (The University of Hong Kong)

Let p be a polynomial in $R[x, y]$. p is a coordinate of $R[x, y]$ if there is an automorphism φ of $R[x, y]$ such that $\varphi(x) = p$. A tame automorphism of $R[x, y]$ is an automorphism which is a composition of affine and elementary automorphisms of $R[x, y]$. p is a tame coordinate of $R[x, y]$ if there is a tame automorphism φ of $R[x, y]$ such that $\varphi(x) = p$. A coordinate which is not tame is called wild. In this talk, the speaker will introduce algorithms to determine coordinates, tame and wild coordinates in $R[x, y]$.

Verbal Subgroups and Subalgebras in Skew Fields

A. Lichtman (University of Wisconsin-Parkside)

An example of a skew field D such that every element of D can be represented as an additive commutator, that is for every element $a \in D$ two elements $b, c \in D$ can be found such that $a = [b, c] = bc - cb$, was constructed first by Harris (see B. Harris, Proc. Amer. Math. Soc. 9 (1958), 628-630). We will consider in this paper the following more general problem. Let R be a (skew) field, K be a central subfield of R , $A(y_1, y_2, \dots)$ be free associative algebra over K with a free countable system of generators $y_1, y_2, \dots,$ L be its free Lie subalgebra generated by the elements y_i $(i = 1, 2, \dots)$. If $w = w(y_{i_1}, y_{i_2}, \dots, y_{i_n})$ is an arbitrary non-zero element of L then the values of w on the algebra R are defined in a natural way and these values form a subset $w(R)$ of R. Further let F be a free group with a free system of generators $x_1, x_2, \dots$ and v be an arbitrary non-unit element of F.

Theorem 1. *Let D be an arbitrary (skew) field with a central subfield K. Then it can be imbedded in a skew field Δ which has the same cardinality as D if D is infinite, and Δ is countable if D is finite, and i) $w(D) = D$ for every non-zero element $w \in L$. ii) $v(\Delta^*) = \Delta^*$ for every non-unit element $v \in F$.*

Corollary. *Let D be an arbitrary (skew) field. Then it can be imbedded in a skew field Δ such that every element of Δ is an additive commutator and every non-zero element is a multiplicative commutator.*

On the two-dimensional subalgebras of polynomial algebras

L. Makar–Limanov (Wayne State University)

Let $A = F[x_1, x_2, ..., x_n]$ be an algebra of polynomials in n variables over a field K and let a, b be a pair of algebraically independent elements of A. In the recent preprint of I. Shestakov and U.Umirbaev, *"Poisson brackets and two-generated subalgebras of rings of polynomials"* the authors obtained lower estimates for the homogeneous degree of a polynomial $p(a, b)$ in terms of the degrees of p relative to the first and second variable and the degrees of a and b, if the characteristic of K is zero. In the talk I'll outline how to obtain these estimates in a way which allows generalizing them to a field of any characteristic and to free associative algebras, as well as to non-homogeneous degrees.

The author was supported by an NSA grant while working on this project.

Generic fibrations by $\boldsymbol{A}\mathbf{1}$ and $\boldsymbol{A}^*$ over discrete valuation rings

N. Onoda (Fukui University)

Let $(R, \pi R)$ be a discrete valuation ring with residue field k and quotient field K and let A be an affine domain over R. Then we say that A is a generic $\boldsymbol{A}^1$-fibration (resp. generic $\boldsymbol{A}^*$-fibration) over R if the generic fiber of A over R is K-isomorphic to the polynomial ring (resp. Laurent polynomial ring) in one variable over K.

Theorem. Suppose that A is normal.
(1) If A is a generic $\boldsymbol{A}^1$-fibration over R, then there exists a reduced Artin ring L such that

$$A/\sqrt{\pi A} \cong_k L[X].$$

(2) If A is a generic $\boldsymbol{A}^*$-fibration over R, then there exists a reduced Artin ring L such that

$$A/\sqrt{\pi A} \cong_k L[X] \times B,$$

where $B = 0$, the null ring, or $B = k[X, X^{-1}]$ or $B = k[X, Y]/(XY)$.

The theorem is concerned with a family of fibers S_a of a surjective morphism $\rho\colon S \to \boldsymbol{A}^1$, where S is a normal affine surface defined over $\boldsymbol{C}$. For example, if $S_a \cong \boldsymbol{A}^1$ for every $a \neq 0$, then $(S_0)_{\mathrm{red}}$ is a disjoint union of affine lines.

The monodromy group of a generic curve covering $\mathbb{P}^1$

T. Shaska (University of California at Irvine)

Let $\mathcal{X}_g$ be a generic curve of genus g and $\phi : \mathcal{X}_g \to \mathbb{P}^1$ a degree n covering. What are the possible groups that can occur as monodromy group of this covering? Zariski showed that for $g > 6$ such monodromy group is not solvable. We continue on work of Fried and Guralnick and discuss cases $2 \leq g < 6$.

The tame and wild polynomial automorphisms

I. Shestakov (University of Sao Paulo)

Not supplied.

Pseudorandom Walks on Elliptic Curves

I. Shparlinski (Macquarie University)

Let $\mathbf{E}$ be an elliptic curve over $\mathbf{F}_q$ give by an affine Weierstrass equation. For a point $G \in \mathbf{E}(\mathbf{F}_q)$, we consider sequences of points

$$W_n = eW_{n-1}, \qquad n = 1, 2, \ldots,$$

where $W_0 = G$, and

$$U_n = U_{n-1} \oplus G, \qquad n = 1, 2, \ldots,$$

where $U_0 \in \mathbf{E}(\mathbf{F}_q)$. We show that from several points of view these sequences have good pseudorandom properties: typically they have a long period, are uniformly distributed and avoid certain structural linearities.

Joint work with Florian Hess and Tanja Lange.

Symmetric matrices, Invariants and the Jacobian Conjecture

A. van den Essen (University of Nijmegen)

In this talk (which is joint work with S.Washburn) it is shown that for all $n \leq 4$ the Jacobian Conjecture holds for all complex polynomial maps $F : C^n \to C^n$ of the form $F = x + H$, where H is homogeneous of degree greater than one and JF is symmetric. It is also shown that the analogues statement for real polynomial mappings $F : R^n \to R^n$ holds for all n.

Trivial, Locally Trivial, and Proper G_a-Actions on Affine n-space

P. van Rossum (New Mexico State Univerisity)

It is well know that on affine 3-space every proper G_a-action is in fact a translation (Deveney, Finston, Gehre, 1994); in fact a recent result (Kaliman, 2003) even shows that every fixed point free action on 3-space is a translation. On affine 5-space, there is an example of a proper G_a-action that is not a translation, in fact that is not even locally trivial (Deveney, Finston, 1995). There is also an example of locally trivial G_a-action on affine 5-space that is not a translation (Winkelman, 1990). This talk will address the missing case of affine 4-space.

Jacobian Relations and Formal Inverse

D. Wright (Washington University)

We explore some methods for showing the formal inverse of a polynomial map F is again a polynomial when F has Jacobian determinant 1. We assume F is a map from affine n-space to itself having the form $z - H$ with H homogeneous of degree $d \geq 2$. The famous Jacobian Conjecture (JC) is reduced to the case $d = 3$. Viewing H as a generic form, the Jacobian hypothesis $\det JF = 1$ is expressed as polynomial constraints on the "coefficients" of H, which are now seen as indeterminates over $\mathbb{Q}$. These Jacobian relations have a combinatoric interpretation ("loop formulas") in the spirit of known combinatoric expressions ("tree formulas") for the formal inverse coefficients. According to Hilbert's Nullstellensatz and the known degree bound for the inverse of a polynomial map, the JC is equivalent to showing the inverse coefficients beyond degree d^{n-1} lie in $\sqrt{\mathcal{J}}$, where $\mathcal{J}$ is the ideal generated by the Jacobian relations. Specific cases can be checked by computer using Groebner bases, but the large number of indeterminates imposes size/time limitations. The quest is to uncover a combinatorial relationship between loops and trees which implies this ideal membership, but we need to know how strong a statement one should expect. For example@: Is $\sqrt{\mathcal{J}}$ really needed, or is $\mathcal{J}$ sufficient? Successful computations have shown that certain inverse coefficients beyond the degree bound lie in $\sqrt{\mathcal{J}}$ but <u>not</u> in $\mathcal{J}$. They also hint toward a positive answer in the cubic linear situation.