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Abstracts

Session 11

Control and Geometric Mechanics

Organizers:

Manuel de León (CSIC)
Alberto Ibort (Universidad Carlos III de Madrid)
Francesco Bullo (Coordinated Science Lab.)

Index of Abstracts

(In this index, in case of multiple authors, only the speaker is shown)

On the global stabilization of the inverted pendulum

Javier Aracil (Universidad de Sevilla)

In this presentation, we deal with the problem of the global stabilization of the inverted pendulum. This problem has attracted much interest in the literature, and has been used as a benchmark for studies in nonlinear control. As a matter of fact, a widely accepted solution has not been obtained yet. The challenging goal is to determine a single smooth feedback law that simultaneously solves two problems: swing-up the pendulum from any initial position (including the downwards) and local stabilization in the upright position. In the presentation a new approach to the problem is proposed, which leads to a family of feedback laws producing the desired behavior. The laws presented are smooth and are obtained with the Hamiltonian energy-shaping methodology. They are based on choosing an appropriate desired potential energy function and a damping term that changes its sign, giving rise alternatively to damping or to energy injection, which might generate undesirable oscillating phenomena. The second part of the presentation is devoted to the problem of designing controllers for generating autonomous oscillations in the inverted pendulum.

Trajectory Design for Mechanical Control Systems: from Geometry to Algorithms

Francesco Bullo (Coordinated Science Lab)

Motion planning and control are key problems in a collection of robotic applications including the design of autonomous agile vehicles and of minimalist manipulators. These problems can be accurately formalized within the language of affine connections and of geometric control theory. This talk presentd recent results on kinematic controllability and on oscillatory controls. The presentation will emphasize how to translate geometric controllability conditions into algorithms for generating and controlling trajectories.

Lie groups and control theory

José Cariñena (Universidad de Zaragoza)

We will present some application in control theory of Lie systems of differential equations admitting a non-linear superposition rule. The reduction theory and connections associated to such system will be shown to play a relevant role.

Covariant Poisson reduction: first steps

Marco Castrillón López (Universidad Complutense de Madrid)

The theory of Poisson reduction is one of the most important issues in geometric Mechanics with deep implications in symplectic geometry, dynamical systems, etc... The root of this procedure is the cotangent reduction. Namely, we have a group G of symmetries of a given Hamiltonian $H : T^*Q \to \mathbb{R}$ and the canonical symplectic structure of T^*Q. In this case the manifold T^*Q/G is not symplectic but Poisson and the Poisson bracket plays a crucial role in the description of the reduced system. When one is dealing with Field Theories, the reduction process described above is not so well understood. In this case we have a bundle $E \to M$ and a Hamiltonian defined on the polysimplectic bundle Π, invariant under a group of symmetries. The goal of this contribution is to twofold. First, we study the relation of the multisymplectic formulation of Hamiltonian systems and the Poisson bracket. Secondly, for principal bundles $E = P \to M$ and Hamiltonian systems invariant under the structure group G, we study the reduction of the Poisson bracket and the equations obtained. This case can be understood as the covariant generalization of the so called Lie–Poisson equations.

Nonholonomic Systems with Symmetry: Some Recent Results and Open Questions

Hernán Cendra (Universidad Nacional del Sur, Argentina))

The methods of symmetry in mechanicas reveal deep geometric facts, with consequences in the local and global dynamical behavior of the system. The connection between the Lagrangian and Eulerian variables in fluid mechanics is analogous to the connection between the, say, the Euler angles and the body angular velocity for the rigid body, a fact clearly described by the modern theory of reduction. The modern era of reduction began with the pioneering work of Arnold on fluid mechanics, 1966, and Smale, on the topology of mechanical systems, 1970. The geometric study of nonholonomic systems reveals similar facts. In this talk, I will explain some recent results, showing the connection between reduced and nonreduced variables for nonholonomic systems. I will emphasize the method of reducing the Lagrange-d´Alembert Principle, rather than reducing the equations. This gives in a very natural way the reduced equations, called Lagrange-Poincaré-d´Alembert Equations.

Gradient control systems

Jorge Cortes* (University of Illinois at Urbana-Champaign)

P. E. Crouch (Arizona State University)

A. J. van der Schaft (University of Twente)

We investigate necessary and sufficient conditions under which a general nonlinear affine control system with outputs can be written as a gradient control system corresponding to some Riemannian metric defined on the state space. The results rely on a suitable notion of compatibility of the system with respect to a given affine connection, and on the output behavior of the prolonged system and of the gradient extension. The symmetric product associated with the affine connection plays a key role in the discussion.

Cartan's approach applied to nonholonomic mechanics

Kurt M. Ehlers (Kurt University)

In this talk we discuss Cartan's approach to problems in nonholonomic mechanics. Consider a free particle $c(t)$ moving with kinetic energy T in a configuration manifold M nonholonomically constrained to a distribution $\mathcal{E}$. Its trajectory is then a nonholonomic geodesic where the geodesic equations are obtained by computing the acceleration $\nabla_{\dot{c}}\dot{c}$ and orthogonally projecting the result onto $\mathcal{E}$. Here ∇ is the Levi-Civita connection associated to T. In Cartan's approach this problem is reformulated in terms of a sub-bundle B of the frame bundle F^*M whose sections are adapted to T and $\mathcal{E}$. We use Cartan's method of equivalence to study the underlying geometry in the case of contact distributions on three-manifolds and Engel distributions on four-manifolds. We also discuss how Cartan's methods can be used as an operational system for studying specific examples such as a rolling/spinning coin.

Cartan forms for first order constrained variational problems

Pedro Luis García Pérez (Universidad de Salamanca)

We stablish a theory of first order constrained variational problems on fibered manifolds. We define a Cartan form for these problems and the corresponding formalism. Finally, we give the relation of this approach with the Theory of Lagrange multipliers.

Quantum optimal control on a compact Riemann manifold with boundary

Alberto Ibort (Universidad Carlos III de Madrid)

We shall describe the problem of optimal control for a quantum free particle moving on a compact Riemannian manifold with boundary by means of quantizing the corresponding classical problem. We will show that the control of the quantum system is exercized by means of the geometry of boundary data and from there we will derive a maximum principle for the solution of the problem.

Planar Propulsion through the Manipulation of Circulatory Flows

Scott D. Kelly (University of Illinois at Urbana-Champaign)

The development of thrust forces on the control surfaces of a variety of biomorphic underwater vehicles corresponds to the manipulation of circulatory flows about these surfaces. Building on reduced Lagrangian models for locomotion through inviscid fluids in the absence of circulation, we examine circulation-based propulsion as it pertains to two archetypical systems. First we present a Hamiltonian model for planar carangiform locomotion in which the strength and relative position of a trailing point vortex are controlled to propel a rigid body; the flow around the vortex in this case approximates the flow around an oscillating hydrofoil. We then examine controllability and motion-planning for the underactuated nonlinear control system comprising an aquatic Flettner rotor with planar mobility.

Flagellar Motion via Geometric Mechanics

Gerusa A. Araujo (LNCC Brazil)

Jair Koiller* (Fundação Getulio Vargas RJ/Brazil)

Flagellar motion was first studied by G.I.Taylor and J.Lighthill in the 1950s. Around 1975 the literature in the field was of order 103 but interest somehow subsided from 1980-2000. Recently, due to new biochemical techniques and optical devices such as the laser tweezer, there is a renewed surge of interest. In this talk we will reexamine flagellar motion from the viewpoint of nonholonomic geometric control theory. Our toy model is a organism consisting of a spherical cell and n+1 segments articulated in n hinges θ_j. For simplicity, we assume planar motion, with (x, y) the coordinates of the center of the cell, and ϕ the angle of the first segment with the x-axis. A motion plan is specified by periodic functions $\theta_j(t), 1 \leq t \leq T$. Due to the zero Reynolds number hidrodynamics, there is a $3 \times N$ connection matrix A relating the velocities $\dot{(\theta)}_j$ with the lie algebra elements $g^{-1}\dot{g}$ where $g(t)$ is the rigid motion associated to (x, y, ϕ). We show how the connection A and its curvature can be computed algorithmically, and we discuss the optimal control problem of maximazing say, the x-displacement, given a fixed energy budget E, under the self-propelling constraint of zero total force and torque.

The role of controllability in motion planning for affine connection control systems

Andrew D. Lewis (Queen's University)

An affine connection control system is defined on a configuration space Q with an affine connection ∇, and is governed by the differential equation

$$\nabla_{\gamma'(t)}\gamma'(t) = \sum_{a=1}^{m} u^a(t) Y_a(\gamma(t)).$$

This system class includes a large number of applications, and also possesses an extremely rich geometric structure.

We shall explore the relationship between low-order controllability and motion planning for such systems. That there is an explicit link between these two topics has only recently been made clear. Remarkably, the idea the ties together low-order controllability and motion planning is a vector-valued quadratic form that one can associate with the system.

One of the interesting developments of this work has been the identification of a large number of examples whose controllability can be determined at low-order, and which are, as a consequence, amenable to certain easily understood, explicit motion planning strategies. Such examples include models for a hovercraft and an underwater vehicle, and the snakeboard. We shall also consider some examples that fail to satisfy the controllability conditions.

Generalized Hamiltonian equations and Optimal Control

Carlos López* (Universidad de Zaragoza)

María de Luna (Zaragoza)

The geometric formulation of Lagrangian and Hamiltonian Classical Mechanics and Classical Field Theory has been a subject of uninterrupted interest during the last decades, partially as a foundation for the program of geometric quantization, but also because of the increasing interest in constrained variational calculus, and its applications to Optimal Control. A unified treatment of both Lagrangian and Hamiltonian approaches to variational calculus has been developed since the work of Skinner and Rusk, first in classical dinamical systems, and then in multivariate variational calculus, clarifying the relationship between both Lagrangian and Hamiltonian formulations in the singular case. A new perspective about the generalized Hamiltonian system for variational calculus has been introduced. At the same time, the constrained variational calculus has a direct relationship with the Optimal Control theory, and therefore, Pontryagin Maximum principle finds in this framework a geometric formulation, closely related to the idea of generalized Hamiltonian system.

On separation of variables for algebraically integrable Hamiltonian systems

Franco Magri (University of Milano-Bicocca)

An algorithm for finding the separation of variables of certain classes of algebraically integrable Hamiltonian systems is presented as an elaboration of the method used by Sophie Kowalewski for her top. According to the method, a certain ideal of polynomials is associated to the systems. The structure of this ideal allow to ascertain the existence of the separation variables in certain (sufficiently wide class), and to find them.

Geometric Integrators in Constrained Mechanics

David Martín de Diego (Instituto de Matemáticas y Física Fundamental, CSIC)

An unified geometric construction of various Geometric Integrators for regular and singular lagrangians with holonomic or non-holonomic constraints is presented. It is essentialy based in the classical technique of generating functions and in the Dirac classification of constraints in first and second class for constrained systems and its extension to nonholonomic systems.

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Lie algebroids and control theory

Eduardo Martínez Fernández (Universidad de Zaragoza)

A structure of Lie algebroid on a vector bundle can be thought as a replacement of the tangent bundle to the base manifold. It will be shown how this structure can be used in geometric mechanics and control theory.

Integrability properties of chained systems in non-holonomic mechanics

Alfonso Anzaldo-Meneses (Universidad Autónoma Metropolitana-Azcapotzalco, México)

Felipe Monroy-Pérez* (Universidad Autónoma Metropolitana-Azcapotzalco, México)

A chained system in $\mathbb{R}^{n+3}$ is defined by a regular distribution $\Delta = \{Z_1, Z_2\}$ with $Z_1 = \partial x_1 - \sum_{k=1}^{n+1} x_{k+1}\partial x_{k+2}$ and $Z_2 = \partial x_2$. A number of mechanical systems with non-holonomic constraints can be formulated by means of $\Delta-$horizontal curves, that is, absolutely continuous curves $t \mapsto q(t)$ satisfying $\dot{q}(t) \in \Delta(q(t))a.e.$ The base space $\mathbb{R}^{n+3}$ can be endowed with a group law for which Δ becomes a left invariant nilpotent basis.

We study the problem of finding the $\Delta-$horizontal curves $t \mapsto q(t)$ which further minimize the energy $\Lambda(g) = \int\langle \dot{g}, \dot{g}\rangle$. We approach the problem as a left invariant optimal control problem. Using the symplectic structure of the group $\mathbb{R}^{n+3}$, we write Hamilton equations in non canonical coordinates $(g, p) \in T^*\mathbb{R}^{n+3}$. The system Hamiltonian turns out to be quadratic $\mathcal{H}(g, p) = p(Z_1(g))^2 + p(Z_p(g))^2$, and using the nilpotency of the corresponding Lie algebra, we provided a recurrence formula for the Casimir elements that guarantee the integrability of the system in terms of an hyper elliptic curve $y^2 = p(x)$.

Control and kinematical systems

Miguel C. Muñoz-Lecanda* (Technical University of Catalonia)

J. Yániz (Technical University of Catalonia)

The aim of this work is to analyze the equivalence between the second order equations describing the dynamics of mechanical systems, and the asociated kinematic system when dealing with nonholonomic systems with controls, and mechanical systems with symmetry.

In the first case, if the system is fully actuated, both systems are equivalent. However, if it is underactuated we must force an extra condition to ensure that a weak equivalence holds. In the second case, the system is reduced to a nonholonomic one, and the above equivalence theorems are applied.

Furthermore, the notion of decoupling vector fields is generalized to a distribution of vector fields. This point of view may be used to obtain better solutions when a cost function is added to the controllability problem. The results are applied to describe properties of some mechanical systems.

A distribution theoretical approach to reduction and Hamiltonian conservation laws

Juan-Pablo Ortega (Institut Non Lineaire de Nice. CNRS.)

In this talk we will see how the use of distribution theoretical techniques is able to capture in a very efficient way the conservation laws associated to the symmetries of a Hamiltonian system encoded in a canonical Lie group action that leaves the system invariant. Our point of view will lead to a presentation of the so called optimal momentum map and of its use in the implementation of symmetry reduction of Poisson and symplectic manifolds.

Passivity based control of mechanical systems

Romeo Ortega

Interconnection and Damping Assignment Passivity–based Control is a technique which achieves stabilization of nonlinear systems assigning a desired (port–controlled Hamiltonian) structure to the closed–loop. Since the introduction of this controller design methodology four years ago many theoretical extensions and practical applications have been reported in the literature. The theoretical developments include some useful variations and shortcuts that may be introduced when the technique is applied to particular classes of systems and the incorporation of additional features to handle control scenarios other than just stabilization. The purposes of this paper are to collect and present in a unified way some of the new theoretical results and to discuss the current research and future directions.

Constrained Poisson systems

Víctor Planas-Bielsa (Institut non lineair de Nice CNRS)

The aim of this work is describe some properties of a class of dynamical systems in which a Poisson flow is constrained to a given subset of the Poisson manifold. The use of such study is justified by the fact that these equations describe, for instance, mechanical systems with nonholonomic constraints, and it is specially focused in that case that these constraints are affine in velocities instead of linear. We also give a Noether theorem for these systems when a Lie group action is a symmetry of the problem.

Quasi-bi versus bi-quasi Hamiltonian systems

Willy Sarlet (Ghent University, Belgium)

We recall the concept of a quasi-bi-Hamiltonian system on a manifold with a double Poisson structure and the use of this double structure for establishing complete integrability. A quasi-bi-Hamiltonian system has a standard Hamiltonian representation with respect to one Poisson structure and a quasi-Hamiltonian representation (i.e. Hamiltonian up to a factor) with respect to the other. Bi-quasi Hamiltonian systems are introduced as those which have a double quasi-Hamiltonian representation and satisfy a further compatibility condition which has to do with the compatibility of Poisson structures on an extended space. It will be shown how such systems also give rise to an algorithm for constructing integrals in involution, leading to complete integrability under certain circumstances. A number of examples will be discussed.

Homogeneous Lagrangian systems

David Saunders (Open University, UK)

The motivation for this talk is a comparison between two straightforward problems in the calculus of variations:

- find the shortest curve joining two points in space;
- find the trajectory in space of a free particle of unit mass.

Although the solutions of both problems are straight lines, there is an important difference between the two cases. For the geometric problem, the solutions do not depend on the parametrization: the property of being 'shortest' depends only on the image of the curve. On the other hand, solutions of the physical problem are trajectories rather than image curves, because in this case the time parameter is an essential part of the problem. These differences appear also in properties of the Lagrangian: for the geometric problem, the Lagrangian is a homogeneous function, whereas for the physical problem the Lagrangian is more properly considered as a differential form. In this talk I shall describe some consequences of these differences, and show how the homogeneous formulation can also be used to explain some features of the physical problems.