NUMERICAL ASYMPTOTIC BEHAVIOR OF A TUMOR CELL MODEL WITH ACTIVE MOTION TOWARD A HELE–SHAW-LIKE MODEL

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Abstract. Let Ω be a connected, open, bounded set of $\mathbb{R}^d$, with $d = 2$ or 3, and $[0, T]$ a time interval. The tumor model that is considered herein [2] governs the cell motion under pressure and active mechanics via a cell population density function $n : \overline{\Omega} \times [0, T] \to \mathbb{R}^+$ satisfying

$$\partial_t n - \nabla \cdot (n \nabla p(n)) - \nu \Delta n = n\, G(p(n)) \quad \text{in } \Omega \times (0, T), \tag{1}$$

subject to the (natural) boundary condition

$$\nabla n \cdot \boldsymbol{n} = 0 \quad \text{on } \partial\Omega \times (0, T), \tag{2}$$

with $\boldsymbol{n}$ being the outward unit normal vector on the boundary $\partial\Omega$, and the initial condition

$$n|_{t=0} = n^0 \quad \text{in } \Omega. \tag{3}$$

Here $p : [0, +\infty) \to [0, +\infty)$ is the pressure defined by

$$p = p(n) := \frac{k}{k-1} n^{k-1} \quad \forall\, n \geq 0, \quad (k \in \mathbb{N},\ k \geq 2),$$

and $G = G(p)$ is a truncated decreasing function such that there exists $P_{\max} > 0$ (the homeostatic pressure) with

$$G(0) > 0, \quad G(p) = 0 \quad \forall\, p \geq P_{\max} > 0, \quad \text{and} \quad G'(p) < 0 \quad \forall\, p \in (0, P_{\max}).$$

and the parameter $\nu > 0$ represents the effect of including the active (random) motion of cells.

In this talk we will present a numerical scheme based on a finite element approach for studying the asymptotic behaviour of the numerical approximations to (1)-(3) as $k \to \infty$. The limiting equation is a free-boundary problem of the Hele–Shaw type in the incompressible case where the interface is captured with a leve set function being the limiting density. Some numerical performances will show the stability of the numerical approximations concerning the asymptotic parameter k. See [1] for more details.

References

[1] Guillén-González, F; Gutiérrez-Santacreu, J. V. *From a cell model with active motion to a Hele-Shaw-like system. A numerical approach.* Preprint arXiv:1805.07658.

[2] Perthame, B.; Quirós, F.; Tang, M.; Vauchelet, N., *Derivation of a Hele-Shaw type system from a cell model with active motion*, Interfaces Free Bound. 16 (2014), no. 4, 489–508.

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