Elastography: tumor identification

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Elastography is a medical imaging technique which gives information about the elastic properties of the tissue. Thus, if we associated the health or illness of a tissue with its stiffness, this method can tell us about the presence of disease. One important motivation of elastography comes from cancer detection: roughly speaking, a tumor tissue is 5 to 28 times stiffer than normal soft tissue and, consequently, the resulting deformation after a mechanical action is smaller.

From the mathematical point of view, we are interested in solving inverse problems governed by a system of PDEs (see [1],[3]). In this work we analyze an inverse problem governed by the linear elasticity system.

$$\left\{\begin{array}{ll} u_{tt} - \mathrm{div}\,(Ae(u)) = f, & (x,t) \in Q \\ u = B, & (x,t) \in \Sigma \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & x \in \Omega \end{array}\right. \tag{1}$$

The aim of the work is determining the fourth-order tensor coefficient A for the initial-boundary value problem governed by the linear elasticity system such that its unique solution satisfies the additional boundary condition,

$$Ae(u) \cdot \nu = \Upsilon \text{ on } S \times (0,T), \text{ for a given data } \Upsilon.$$

We reformulate the inverse problem as the minimization of a functional in an appropriate constraint set and we prove that this extremal problem possesses at least one solution with the help of some regularity results (see [2]). Finally we show some numerical experiments in order to check the efficiency of the method solving inverse problems.

Referencias

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