Generalised global supersolutions with mass control for systems with taxis

Anna Zhigun

Queen's University Belfast, School of Mathematics and Physics
University Road, Belfast BT7 1NN, Northern Ireland, UK
e-mail: A.Zhigun@qub.ac.uk

Abstract

In this talk we present a new global solvability concept for the Keller-Segel system

$$\begin{cases} \partial_t u = \nabla \cdot (\nabla u - \chi(u,v) u \nabla v) & \text{in } \mathbb{R}^+ \times \Omega, \\ \partial_t v = \Delta v - v + u & \text{in } \mathbb{R}^+ \times \Omega, \\ \partial_\nu u = \partial_\nu v = 0 & \text{in } \mathbb{R}^+ \times \partial\Omega, \\ u(0,\cdot) = u_0,\ v(0,\cdot) = v_0 & \text{in } \Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^N$,

$$N \geqslant 2.$$

It has long been suspected that in the classical case, i.e. when $\chi \equiv 1$, system (1) possesses solutions which blow-up in finite time. The later means that there are solutions which start as classical at $t = 0$ but exist only up to some finite time $T_{max} \in (0, \infty)$ when the aggregation and collapse of the u-component occur. This hypothesis was finally confirmed [2] for the case when Ω is a ball.

One of the questions naturally arising in connection with the blow-up phenomena is the following: how can a solution be reasonably extended beyond its blow-up time? In answer to this question, we construct [3] a generalised global supersolution with a control upon the total muss. Our result holds for any continuous χ and in arbitrary space dimension. In fact, just as it is the case for the solvability in the classical sense, the upper-triangular structure of system (1) turns out to be a decisive factor for the generalised solvability as well. More precisely, the outcome can be summarised as follows: the classical theory for upper-triangular systems [1] implies that (1) possesses a classical solution as long as it doesn't blow-up, and if it does, then it continuous to exist as generalised [3].

References

[1] H. Amann. "Nonhomogeneous linear and quasilinear elliptic and parabolic boundary value problems." In: *Function spaces, differential operators and nonlinear analysis. Survey articles and communications of the international conference held in Friedrichsroda, Germany, September 20-26, 1992.* Stuttgart: B. G. Teubner Verlagsgesellschaft, 1993, pp. 9–126.

[2] M. Winkler. "Finite-time blow-up in the higher-dimensional parabolic-parabolic Keller-Segel system". In: *J. Math. Pures Appl. (9)* 100.5 (2013), pp. 748–767.

[3] A. Zhigun. "Generalised global supersolutions with mass control for systems with taxis". In: *SIAM Journal on Mathematical Analysis* (accepted). arXiv: 1806.06715.